STEAM into Math Algebra I, Math I and Algebra II Coming Soon!

Algebra I and Algebra II STEAM program for high school students

Did you know that 75% of teens dropping out of high school cite Algebra as the reason they are leaving? It only makes sense that algebra should be taught in a different way to try to reach these students. We cannot teach algebra the same way over and over again and expect different results.

Americans spend $1 billion a year on remedial classes in college that cover material that should have been learned in high school (for free). Most of these remedial classes are mathematics. About 70% of everyone going to a community college test into a remedial math class and most do not pass it.

STEAM into Math is an exciting new program being developed by TPS Publishing through a combination of hands-on projects and lessons designed to bring clarity and support in learning math.
Whether you are taking the traditional route defined by the Common Core, or if you have decided to follow an integrated Mathematics I, II and III course, your students will greatly benefit from using a STEAM math program. The TPS program 'STEAM into Math' includes STEM project content to cover both curricula as can be viewed by the outline overleaf.

STEAM into Math includes:

- **Teacher and student textbook**
- **STEM Real Numbers project book**
- **Interactive Assessment generator or Assessment generator with print license**
- **Online arts projects library for far below grade students**
- **Forensic science with math assignments**

STEM lesson plans are activity- based, where students drill down into the math and together with exciting STEM projects and assessment materials, crime scene math projects, ensures ALL students can access the math. As the teacher, you can choose whether the STEM project should be completed first to offer conceptual understanding or if you prefer to teach students the math in a traditional manner first and then have the STEM as an exciting finale. In the teacher textbook traditional lesson plans, TPS provides:

- **Concepts broken down into clear, step by step instructions for student understanding**
- **Teacher, Parent, Student help sheet provided for teacher, parent, and student support and engagement.**
- **Support lessons provided to help with prior knowledge needed for algebra 1 module**
- **Detailed workings shown for the teacher/parent**
- **Written assessments and online database of practice questions available**

The STEM Projects provide exciting real-life scenarios using the math. CeMaST professors visited over 24 sites looking for the use of algebra. When CeMaST professors found it, they wrote the lessons based on how it was used. For example, since people at a landfill use algebra to calculate compaction rates, we taught it by calculating compaction rates of wadded up paper in a box.

These lessons are taught in a learning cycle format. This format starts with an activity where the students experience the content. This is followed by a discussion that allows them to fully understand what they just experienced and allows the teacher to conduct formative assessment. The teacher can adjust the lesson as necessary at this point. The lesson continues with an application of the concept. Finally, there are several ideas presented for how the content or context can be expanded. For example, after students calculate fuel consumption on a piston engine airplane in gallons per hour, they can do jet engines in pounds per hour. This keeps the faster students productively engaged while slower students finish up.

The STEM projects provide visual assessment by teachers as they watch students follow the DAPIC process. Students define, assess, plan, implement and communicate.

All materials necessary for the projects are readily available and relatively inexpensive, but TPS also provide kits if you prefer to order with the books. No computers or special software is required.

Professional Development is available, although it is not necessary to address the content. The learning cycle format is usually unfamiliar to mathematics teachers and requires some guidance to get started. TPS provide webinars, toll free ongoing help throughout the school year or you can pay for the CeMaST professors to come to your site.

Algebra I and Mathematics I content
As defined by the Common Core Standards

Unit	Title	Content
1	Relationships Between Quantities	• Reason quantitatively and use units to solve problems. • Interpret the structure of expressions. • Create equations that describe numbers or relationships.
2	Reasoning with Equations	• Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Solve systems of equations.
3	Linear and Exponential Relationships	• Extend the properties of exponents to rational exponents. • Solve systems of equations. • Represent and solve equations and inequalities graphically. • Understand the concept of a function and use function notation. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.
4	Descriptive Statistics	• Summarize, represent, and interpret data on a single count or measurement variable. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.

Algebra I and Mathematics I content
As defined by the Common Core Standards

Unit	Title	Content
5 Omit for Math I	Expressions and Equations	• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Solve systems of equations.
6 Omit for Math I	Quadratic Functions and Modeling	• Use properties of rational and irrational numbers. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems.
7 Omit for Algebra I	Congruence, Proof, and Constructions	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Make geometric constructions.
8 Omit for Algebra I	Connecting Algebra and Geometry through Coordinates	• Use coordinates to prove simple geometric theorems algebraically.

Algebra II
As defined by the Common Core Standards

Unit	Title	Content
1	Polynomial, Rational, and Radical Relationships	• Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations. • Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. • Understand solving equations as a process of reasoning and explain the reasoning. • Represent and solve equations and inequalities graphically. • Analyze functions using different representations.
2	Trigonometric Functions	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric function. • Prove and apply trigonometric identities.
3	Modeling with Functions	• Create equations that describe numbers or relationships. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems.

Algebra II
As defined by the Common Core Standards

Unit	Title	Content
4	Inferences and Conclusions from Data	• Summarize, represent, and interpret data on single count or measurement variable. • Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from ample surveys, experiments and observational studies. • Use probability to evaluate outcomes of decisions.

N.CN.1, N.CN.2

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Lesson Plan

Title: Introduction to Complex Numbers

Time required to teach this lesson: 2 x 50 minutes

Vocabulary: Complex, Real Numbers, imaginary Numbers

Materials required for this lesson: Paper, pencils

Concepts:
- Extending knowledge of the number system beyond real numbers into complex numbers
- Using the commutative, associative and distributive properties with complex numbers.

Prior Knowledge:
- Study of the number system including natural numbers, integers, rational
- Use of the commutative, associative and distributive properties with real numbers

Suggested Lesson Plan:

Learning Targets:
- To understand what a complex number is, and understand the difference between the real and imaginary parts.
- To understand how to add, subtract and multiply complex numbers.
- To understand how to use the commutative, associative and distributive properties with complex numbers.

Introduction:
Revise with students the different subsets of numbers, starting with natural numbers, then integers, rational numbers, and real numbers. Discuss with students the word 'real' and ask students what this implies about what they may not have learned about so far. They may see that if there are a set of 'real' numbers, that there must be a set of 'imaginary' numbers.

Revise with students how, when a number is squared, the result is always positive. Show how this means that there is no way that you can square a number and get a negative answer. Then, use this property to show how we cannot within the real number system

N.CN.1, N.CN.2

Lesson Plan

get an answer to the question $\sqrt{-1}$.

Show how we can have a solution to this, that is called an 'imaginary' number:

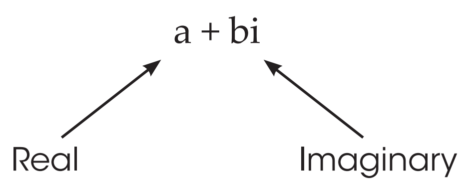
$$\sqrt{-1} = i$$

Middle:
Demonstrate to students some basic properties of complex numbers such as:

$$i \times i = -1$$

$$\sqrt{-a} = \sqrt{a} \times \sqrt{-1} = i\sqrt{a}$$

Show how complex numbers have both a real, and imaginary part. Show how the real part does not have any imaginary (i) part, and imaginary parts have an i part.



Show how here, a and b are real numbers. Show how when a number has both real and imaginary parts, together they make up complex numbers. Explain how here, b is a real number that is multiplied by i.

Revise with students what the commutative, associative and distributive properties are for addition, subtraction and multiplication are, when used with real numbers.

Derive with students the properties when using complex numbers:

<u>Addition:</u>
Commutative: $(a + bi) + (c + di) = (c + di) + (a + bi)$
Associative: $((a + bi) + (c + di)) + (e + fi) = (a + bi) + ((c+di) + (e + fi))$
Distributive: $(a + bi)((c + di) + (e + fi)) = (a + bi)(c + di) + (a + bi)(e + fi)$

<u>Subtraction:</u>
Commutative: $(a + bi) - (c + di) = (c + di) - (a + bi)$
Associative: $((a + bi) - (c + di)) - (e + fi) = (a + bi) - ((c + di) - (e + fi))$
Distributive: $(a + bi)((c + di) - (e + fi)) = (a + bi)(c + di) - (a + bi)(e + fi)$

N.CN.1, N.CN.2

Lesson Plan

Multiplication:
Commutative: $(a + bi) \times (c + di) = (c + di) \times (a + bi)$
Associative: $((a + bi) \times (c + di)) \times (e + fi) = (a + bi) \times ((c + di) \times (e + fi))$
Distributive: $(a + bi) \times ((c + di) + (e + fi)) = (a + bi) \times (c + di) + (a + bi) \times (e + fi)$

Explain to students that this shows that the same commutative, associative and distributive rules apply to complex numbers as they do to real numbers.

Go through examples of addition, subtraction and multiplication with students. Students can now complete the student exercise.

Extension Activity:
Students complete complex number calculations involving squaring and cubing.

Summary:
Ask students to write a paragraph to explain what an imaginary number is, and what a complex number is.

Give examples of complex number calculations and instruct students to figure them out.

Support: Students may benefit from seeing real and imaginary parts of numbers in different colors, to emphasise the difference between them. Students will need additional help in understanding what a complex number is.

ELL: Students will benefit from definitions of key words written in their own language.

N.CN.1, N.CN.2

N.CN.1 Know there is a complex number i such that i2 = –1, and every complex number has the form a + bi with a and b real.
N.CN.2 Use the relation i2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Student Narrative

Natural numbers are whole numbers bigger than zero: 1, 2, 3, 4, …..

Integers are whole numbers that include negatives: … , -4, -3, -2, -1, 0, 1, 2, 3, 4, ….

Rational numbers are numbers that can be written as a fraction with integers as numerator and denominator, for example $\frac{1}{2}$, $\frac{3}{7}$.

Irrational numbers are numbers that cannot be written as a fraction. These numbers will have decimals that never repeat, for example $\pi = 3.14159\ldots\ldots$, $\sqrt{2} = 1.4142135\ldots..$

All of these numbers are, collectively, known as real numbers. You may have heard of this term before. If there are real numbers, does that mean that there are fake numbers?

Not quite – there are what are known as 'imaginary numbers'. It's a bit misleading. Imaginary numbers do 'exist', in the same way that 1, 2, 3 and so on 'exist'. An imaginary number is found by square rooting a negative number. Up until now, you may have thought that squaring negative numbers was not possible. This isn't completely true. Within the real number system it is not possible.

We write imaginary numbers as i, standing for imaginary.

$$\sqrt{-1} = i$$

This also means that:

$$i^2 = -1$$

You will see that the distributive, associative and commutative properties in addition and multiplication apply the same way as with real numbers. In this lesson we look at using imaginary numbers in basic calculations.

N.CN.1, N.CN.2

Important Vocabulary

Complex	A number that consists of both real and imaginary parts
Real	A number that does not have an imaginary part, it can be any number that is rational or irrational
Imaginary	A number that is defined as the square root of -1, written as i

N.CN.1, N.CN.2

TPS Help Sheet

An imaginary number is found by square rooting a negative number. The imaginary number i is defined as the square root of negative 1:

$$\sqrt{-1} = i$$

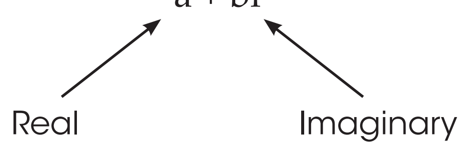
This means that:

$$i^2 = -1$$

Counting up in 1s, real numbers will go ..., -3, -2, -1, 0, 1, 2, 3,...
Counting up in the same way, imaginary numbers will go ..., -3i, -2i, -i, 0, i, 2i, 3i, ...

A complex number has both a real, and an imaginary part:

$$a + bi$$



Where a and b are real numbers.

<u>Examples:</u>

$i + i = 2i$

$(2 + 3i) + (4 + 5i) = 2 + 4 + 3i + 5i = 6 + 8i$

$(5 + 4i) + (3 + 2i) = 8 + 6i$

$(10 + 3i) - (8 + i) = 2 + 2i$

$(-4 - 2i) + (5 - 5i) = 1 - 7i$

When multiplying complex numbers, you need to multiply in the same way that you would with double brackets in algebra:

$(3 + 5i) \times (4 + 7i) = 3 \times 4 + 3 \times 7i + 5i \times 4 + 5i \times 7i$

$= 12 + 21i + 2\text{-}i - 35$

$= -23 + 41i$

N.CN.1, N.CN.2

TPS Help Sheet

Note here that

$$5i \times 7i = 5 \times 7 \times i \times i = 35 \times i = -35$$

Examples:

$$3(2 + 5i) = 6 + 15i$$

$$2i(3 + 4i) = -8 + 6i$$

N.CN.1, N.CN.2

Student Exercise 1

Simplify the following:

1) $(3 + 4i) + (2 + 5i) =$ $5 + 9i$

2) $(5 + 2i) + (7 + 4i) =$ $12 + 6i$

3) $(7 + 4i) + (3 - 5i) =$ $4 - i$

4) $(-4 - 3i) + (-5 + 5i) =$ $-9 + 2i$

5) $(-5 - 2i) + (-4 - 3i) =$ $-9 - 5i$

6) $(5 + 3i) - (2 + i) =$ $3 + 2i$

7) $(8 + 4i) - (4 + 6i) =$ $4 - 2i$

8) $(4 - 2i) - (2 - 5i) =$ $2 + 3i$

9) $(12 + 4i) - (-5 + 4i) =$ 17

10) $(-1 - 2i) - (-1 - 3i) =$ i

11) $(3 + 2i)(4 + 5i) =$ $2 + 23i$

12) $(5 + 3i)(4 + 2i) =$ $14 + 22i$

13) $(5 - 4i)(5 - 3i) =$ $13 - 35i$

14) $4(3 + 2i) =$ $12 + 6i$

15) $-6(4 - 3i) =$ $-24 + 18i$

16) $3i(2 - 4i) =$ $12 + 6i$

17) $-4(3 + i) =$ $-12 - 4i$

18) $-7i \times -5i \times 3i =$ $-105i$

19) $-8i(3 - 2i) =$ $-16 - 24i$

20) $5i \times i \times 3i \times i =$ 15

N.CN.1, N.CN.2

Extension Exercise

Simplify the following:

1) i^2 = -1

2) i^3 = - i

3) i^4 = 1

4) $(2i)^2$ = -4

5) $(1 + i)^2$ = 2i

6) $(1 + i)^3$ = 2i - 2

Comprehension of the Standard

Write a paragraph to explain what an imaginary number is. Explain the difference between an imaginary and a complex number.

Answers will vary

Work out the following:

1) $(3 + 5i) + (2 + 5i)$ = 5 + 2i

2) $(5 + 2i) - (3 - 5i)$ = 8 + 7i

3) $(2 + 5i)(3 - 4i)$ = 26 + 7i

N.CN.1, N.CN.2

Assessment Questions

Simplify the following:

1) $(5 - 3i) + (8 + i) =$ $13 - 2i$

2) $(5 + 4i) + (-3 - 4i) =$ 2

3) $(3 - 3i) - (5 + 7i) =$ $- 2 - 10i$

4) $(7 + 2i) - (10 - 4i) =$ $-3 + 6i$

5) $4i(3 + 2i) =$ $-8 + 12i$

6) $(1 + 3i)(2 + i) =$ $-1 + 7i$

Critical Thinking Homework

How do imaginary numbers look on a number line? Do some research into how complex numbers are shown.
Using your research, show 5 different numbers in this way.

Answers will vary.

N.CN.7

Solve quadratic equations with real coefficients that have complex solutions.

Lesson Plan

Title: Quadratic Equations with Complex Answers

Time required to teach this lesson: 50 minutes

Vocabulary: Solution, Complex, Real

Materials required for this lesson: Paper, pen

Concepts:
- Understanding when a quadratic equation will have real or complex solutions
- Solving quadratic equations with complex solutions

Prior Knowledge:
- Solving quadratic equations with real solutions
- Using the quadratic formula
- Knowledge of the complex number system

Suggested Lesson Plan:

Learning Targets:
- To understand where a solution to a quadratic equation is real or complex
- To understand how to use the quadratic formula to find complex solutions.

Introduction:
Revise with students what a quadratic equation is and revise how to solve these equations. Use a variety of examples where students can solve the equations by factoring, as well as by using the quadratic formula and completing the square.

Students can complete exercise 1.

Middle:
Discuss with students how they will have come across quadratic equations that seemingly have no solutions. Discuss with students where this may have occurred. Give students the following question to solve:

$$x^2 + 4x + 5 = 0$$

N.CN.7

Lesson Plan

Discuss solving this equation with students. It will not factor, so students either need to complete the square or use the quadratic formula. Show, by using the formula, that the following will occur:

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

Discuss how, with pervious knowledge, we would say there are 'no real solutions'. However, with new understanding of the complex number system, we can find complex solutions by finding the square root of -4 to be $2i$ or $-2i$.

Instruct students to solve this. Students should get answers of:

$$x = -2 + i \text{ or } -2 - i$$

Show how, by substituting these answers into the original equation, we have two solutions:

$$x = -2 + i:$$
$$(-2 + i)^2 + 4(-2 + i) + 5 =$$
$$4 - 4i - 1 - 8 + 4i + 5 = 0 \checkmark$$

$$x = -2 - i:$$
$$(-2 - i)^2 + 4(-2 - i) + 5 =$$
$$4 + 4i - 1 - 8 - 4i + 5 = 0 \checkmark$$

Discuss with students how they can determine if an equation has a complex solution. Students should see that his occurs where the square root is negative, meaning that:

$$b^2 - 4ac < 0$$

Show students how to solve the equation above by completing the square:

$$x^2 + 4x + 5 = 0$$
$$x^2 + 4x = -5$$
$$(x + 2)^2 - 4 = -5$$
$$(x + 2)^2 = -1$$
$$x + 2 = \pm\sqrt{-1}$$
$$x + 2 = i \text{ or } x + 2 = -i$$
$$x = -2 + i \text{ or } x = -2 - i$$

N.CN.7

Lesson Plan

Students can now complete exercise 2.

Extension Activity:
Students determine if equations have real or complex solutions without solving.

Summary:
Instruct students to write a paragraph to explain when a quadratic equation will have a complex solution. Give students examples of quadratic equations and have them determine if the solutions are real or complex, and find the solution.

Support: Students will need extensive recapping of how to solve quadratic equations with real solutions before looking at complex solutions.

ELL: Students will benefit from diagrams to show how complex solutions can be found to quadratic equations, by square rooting the negative as appropriate.

N.CN.7

Solve quadratic equations with real coefficients that have complex solutions.

Student Narrative

You will have solved many quadratic equations in the past. Sometimes, you will have come across times where you were not able to find a solution. For example, look at the following equation:

$$x^2 + 4x + 5 = 0$$

It will not factor, so to solve it you will need to either complete the square or use the quadratic formula.

You will find when you do either of these that you come across a negative square root. In the past, this is where you would stop and say there are 'no real solutions'. However, there are complex solutions.

Important Vocabulary

Solution	An answer to a question, a number
Complex	A number that consists of both real and imaginary parts
Real	A number that does not have an imaginary part, it can be any number that is rational or irrational

N.CN.7

TPS Help Sheet

A quadratic equation is one that has the form:

$$ax^2 + bx + c = 0$$

Where a,b,and c are constants. A quadratic equation will almost always have two solutions for x, that can be found by different methods – factoring, completing the square, or using the quadratic formula.

Sometimes, there are no 'real' solutions to quadratic equations. That is, there will be a complex solution because at some point you will need to square root a negative number. These can be solved by completing the square or using the quadratic formula:

Example: $x^2 + 8x + 20 = 0$

Completing the square:

$$x^2 + 8x + 20 = 0$$
$$(x + 4)^2 - 16 + 20 = 0$$
$$(x + 4)^2 + 4 = 0$$
$$(x + 4)^2 = -4$$
$$x + 4 = \sqrt{-4}$$
$$x + 4 = 2i \text{ or } x + 4 = -2i$$
$$x = -4 + 2i \text{ or } x = -4 - 2i$$

Quadratic formula:

Remember that, for any quadratic

$$ax^2 + bx + c = 0$$

We have that:

$$x = \frac{-b \pm \sqrt{b2 - 4ac}}{2a}$$

For this problem

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 20}}{2\times1}$$

$$x = \frac{-8 \pm \sqrt{-16}}{2}$$

$$x = \frac{-8 \pm 4i}{2}$$

$$x = -4 + 2i \text{ or } x = -4 - 2i$$

N.CN.7

TPS Help Sheet

You can determine if a solution will be complex if, using the quadratic formula:

$$b^2 - 4ac < 0$$

Student Exercise 1

Solve, by any means, these quadratic equations.

1) $x^2 - 9x + 20 = 0$ $x = -4$ or -5

2) $x^2 - 9x + 18 = 0$ $x = -3$ or -6

3) $x^2 + x - 20 = 0$ $x = -5$ or 4

4) $x^2 + 5x - 6 = 0$ $x = -1$ or 6

5) $x^2 - 7x + 6 = 0$ $x = -6$ or -1

6) $x^2 + 3x - 10 = 0$ $x = -5$ or 2

7) $x^2 + 9x + 14 = 0$ $x = -7$ or -2

8) $x^2 - x + 20 = 0$ $x = -4$ or 5

Student Exercise 2

Solve, by any means, these quadratic equations.

1) $x^2 + 6x + 13 = 0$ $x = -3 + 2i$ or $x = -3 - 2i$

2) $x^2 - 4x + 13 = 0$ $x = 2 + 3i$ or $x = 2 - 3i$

3) $x^2 + 8x + 20 = 0$ $x = -4 + 2i$ or $x = -4 - 2i$

4) $x^2 + 10x + 50 = 0$ $x = -5 + 5i$ or $x = -5 - 5i$

5) $x^2 + 12x + 37 = 0$ $x = -6 + 5i$ or $x = -6 - 5i$

6) $x^2 + 14x + 53 = 0$ $x = -7 + 2i$ or $x = -7 - 2i$

7) $x^2 + 10x + 41 = 0$ $x = -5 + 3i$ or $x = -5 - 3i$

N.CN.7

Student Exercise 2

8) $x^2 - 6x + 10 = 0$ $x = 3 + i$ or $x = 3 - i$

9) $2x^2 + 3x + 6 = 0$ $x = -0.75 + 1.56i$ or $x = -0.75 - 1.56i$

10) $5x^2 + 4x + 10 = 0$ $x = -0.4 + 1.36i$ or $x = -0.4 - 1.36i$

11) $4x^2 + 3x + 5 = 0$ $x = -0.375 + 1.05i$ or $-0.375 - 1.05i$

12) $5x^2 - 6x + 5 = 0$ $x = 0.6 + 0.8i$ or $x = 0.6 - 0.8i$

Extension Exercise

Without solving these equations, determine if the solutions are real or complex.

1) $x^2 + x + 1 = 0$ Complex

2) $x^2 - x - 1 = 0$ Real

3) $x^2 + 5x - 5 = 0$ Real

4) $x^2 + 3x + 5 = 0$ Complex

Comprehension of the Standard

Write a paragraph to explain what makes a quadratic equation have a real or complex solution.

Answers will vary

For each of these, solve the equations.

1) $x^2 - 10x + 26 = 0$ $x = 5 + i$ or $x = 5 - i$

2) $2x^2 + 2x + 5 = 0$ $x = -0.5 + 1.5i$ or $x = -0.5 - 1.5i$

N.CN.7

Assessment Questions

1) $x^2 - 2x + 2 = 0$ $x = 1 + i$ or $x = 1 - i$

2) $x^2 + 6x + 27 = 0$ $x = -3 + 4i$ or $x = -3 - 4i$

3) $3x^2 + 2x + 7 = 0$ x = −13 + 1.49i or x = −13x = -0.333...+ 1.49i or x = -0.333...- 1.49i

4) $2x^2 - 5x + 10 = 0$ $x = 1.25 + 1.85i$ or $x = 1.25 - 1.85i$

Critical Thinking Homework

What other kinds of equations can have complex solutions?

Can cubic equations have complex solutions?

Can linear equations have complex solutions?

Do some research into where complex numbers are applied in math.

Answers will vary.

N.CN.8, N.CN.9

Extend polynomial identities to the complex numbers.
For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Lesson Plan

Title: Using Complex Numbers in Polynomials

Time required to teach this lesson: 2 x 50 minutes

Vocabulary: Polynomials, Complex Number

Materials required for this lesson: Paper, pen

Concepts:
- Understanding polynomial identities using complex numbers
- Using polynomial identities to factorize polynomials
- Understanding the Fundamental Theorem of Algebra

Prior Knowledge:
- Using complex numbers in addition, subtraction, multiplication and division
- Understanding polynomial identities using real numbers

Suggested Lesson Plan:

Learning Targets:
- To understand polynomial identities using complex numbers
- To understand how to apply polynomial identities with complex numbers
- To understand the Fundamental Theorem of Algebra

Introduction:
Write the following real expressions, and have students write them as equivalent expressions:

$(a + b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a + b)(c + d) = ac + ad + bc + bd$

$a^2 - b^2 = (a + b)(a - b)$

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Discuss with students how these are polynomial identities, meaning that the rules shown here will work for any values of a and b.

N.CN.8, N.CN.9

Lesson Plan

Middle: Show students the following:

$$a^2 + b^2$$

Show how, using imaginary numbers, we can rewrite this expression in the following way:

$$a^2 + b^2 = a^2 - i^2 b^2$$

Show how this transforms this identity into a difference of squares, meaning that we can write this as

$$(a + ib)(a - ib)$$

Instruct students to expend these to reach $a^2 + b^2$, in order to check that the identity is true.

Show students how, using this technique, we can factor different expressions such as

$$x^2 + 9 = x^2 - 9i^2 = (x + 3i)(x - 3i)$$

Show how, using this, we can find two roots to the equation as follows:

$$(x + 3i)(x - 3i) = 0$$

$$x + 3i = 0 \text{ and } x - 3i = 0$$

$$x = -3i \text{ and } x = 3i$$

Show how, by factoring, we can solve equations such as:

$$x^4 - 64 = 0$$

$$(x^2 + 8)(x^2 - 8) = 0$$

$$(x + \sqrt{8}\, i)(x - \sqrt{8}\, i)(x + \sqrt{8})(x - \sqrt{8}) = 0$$

This means roots are found where:

$$x = -\sqrt{8}\, i,\ x = \sqrt{8}\, i,\ x = -\sqrt{8},\ x = \sqrt{8}$$

Students can now complete exercise 1.

Show student show this leads to the **Fundamental Theorem of Algebra**:

For any polynomial, $p(x) = ax^n + bx^{n-1} + \ldots + z$, there are n roots. In other words, for any polynomial, there are as many roots to the polynomial as the highest power in the polynomial. Show examples of polynomials and have students spot the number of roots in the equation.

N.CN.8, N.CN.9

Lesson Plan

Show students how, using the quadratic formula, they can find always find both roots to quadratic polynomials by applying the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and evaluating the square root for both positive and negative values. Show how there is always a complex root, where real roots can be written with a $+0i$ or $-0i$ component. Show how, since the complex element is calculated with the square root, this will always give an imaginary part with the same magnitude.

Extension Activity:
Students look at finding the all roots to polynomials of higher orders.

Summary:
Show students a series of polynomials, and have them identify the number of roots. Give examples of polynomials of the form $a^2 + b^2$ and instruct them to factor the expressions fully. Assess understanding of key skills.

Support: Students will need to recap key polynomial identities for real numbers, particularly a difference of squares, before moving into complex numbers.

ELL: Students will benefit from examples and instructions written in their preferred language.

N.CN.8, N.CN.9

Extend polynomial identities to the complex numbers.
For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Student Narrative

Polynomial identities are generalised rules that can be applied in helping to solve problems. You will have come across some and applied them within real numbers:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(c + d) = ac + ad + bc + bd$$
$$a^2 - b^2 = (a + b)(a - b)$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

Now that we have the complex number system, we can apply these and find new rules. These new rules will give us more flexibility in what we can do with polynomials.

It will also lead to, what is called the 'Fundamental Theorem of Algebra'

Important Vocabulary

Polynomial	An algebraic expression that includes variables and coefficients that involve only addition, subtraction, multiplication and division
Complex Number	A number that consists of both real and imaginary parts

N.CN.8, N.CN.9

TPS Help Sheet

You will have seen the difference of squares identity with real numbers:

$$a^2 - b^2 = (a + b)(a - b)$$

This allows us to factor the following examples:

$$x^2 - 36 = (x + 6)(x - 6)$$

$$x^2 - 1 = (x + 1)(x - 1)$$

$$x^2 - 5 = (x + \sqrt{5})(x - \sqrt{5})$$

We can exploit the fact that $i^2 = -1$ to transform $a^2 + b^2$:

$$a^2 + b^2 = a^2 - -b^2$$

$$a^2 + b^2 = a^2 - i^2 b^2$$

Here we have a difference of squares, with an imaginary element. We can therefore see that:

$$a^2 + b^2 = (a + ib)(a - ib)$$

We can use this to factor similar expressions such as the following:

$$x^2 + 36 = x^2 + 6^2 = (x + 6i)(x - 6i)$$

$$x^2 + 1 = x^2 + 1^2 = (x + i)(x - i)$$

$$x^2 + 2 = x^2 + (\sqrt{2})^2 = (x + \sqrt{2}\, i)(x - \sqrt{2}\, i)$$

By calling x^2 something else, such as y, in the following, we can simplify as follows:

$$x^4 + 4x^2 - 5$$

Let $y = x^2$. This can be written as:

$$y^2 + 4y - 5 = (y + 5)(y - 1)$$

So:

$$(x^2 + 5)(x^2 - 1) = (x + \sqrt{5}\, i)(x - \sqrt{5}\, i)(x + 1)(x - 1)$$

The **Fundamental Theorem of Algebra** – For any polynomial:

$$p(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots .z$$

There are n complex roots. That is, to find a root, you need to look at the highest power in the polynomial. Note only a real number can be written as a complex number where the complex part is $0i$.

For example,

$p(x) = x^4 + 3x^2 + x - 5$ has 4 roots, as the highest power is 4.

N.CN.8, N.CN.9

TPS Help Sheet

$p(x) = x^5 + 1$ has 5 roots, as the highest power is 5.

For quadratics (ie where the highest power is 2) there are always two roots, that are always two complex roots. You can always find them by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The complex part is found by evaluating $\sqrt{b^2 - 4ac}$

Student Exercise 1

Factor the following:

1) $x^2 + 4 =$ $(x + 2i)(x - 2i)$

2) $x^2 + 9 =$ $(x + 3i)(x - 3i)$

3) $x^2 + 10 =$ $(x + \sqrt{10}\, i)(x - \sqrt{10}\, i)$

4) $x^2 + 5 =$ $(x + \sqrt{5}\, i)(x - \sqrt{5}\, i)$

5) $x^4 + 5x^2 + 6 =$ $(x + \sqrt{2}\, i)(x - \sqrt{2}\, i)(x + \sqrt{3}\, i)(x - \sqrt{3}\, i)$

6) $x^4 + 8x^2 + 12 =$ $(x + \sqrt{6}\, i)(x - \sqrt{6}\, i)(x + \sqrt{2}\, i)(x - \sqrt{2}\, i)$

7) $x^8 - 36 =$ $(x^2 + \sqrt{6}\, i)(x^2 - \sqrt{6}\, i)(x^2 + \sqrt{6})(x^2 - \sqrt{6})$

8) $x^4 - 49 =$ $(x + \sqrt{7}\, i)(x - \sqrt{7}\, i)(x + \sqrt{7})(x - \sqrt{7})$

N.CN.8, N.CN.9

Student Exercise 2

Using the Fundamental Theorem of Algebra, factor all of the following, giving your roots as complex numbers.

1) $x^2 + 5x + 6$ $-2 + 0i, -2 - 0i$

2) $x^2 - 6x + 4$ $3 + \sqrt{5}, 3 - \sqrt{5}$

3) $x^2 - 7x - 8$ $-1 + 0i, 8 + 0i$

4) $x^2 - 2x + 5$ $1 + 2i, 1 - 2i$

5) $x^2 + 10x + 3$ $-5 + \sqrt{22} + 0i, -5 - \sqrt{22} + 0i$

6) $x^2 + 2x + 22$ $-1 + \sqrt{21}\, i, -1 - \sqrt{21}\, i$

7) $x^2 + 4x + 10$ $-2 + \sqrt{6}\, i, -2 - \sqrt{6}\, i$

8) $x^2 + 6x + 16$ $-3 + \sqrt{7}\, i, -3 - \sqrt{7}\, i$

9) $x^2 + 8x + 100$ $-4 + 2\sqrt{21}\, i, -4 - 2\sqrt{21}\, i$

10) $x^2 + 16$ $0 - 4i, 0 + 4i$

Extension Exercise

Find all roots to:

1) $25^4 + 81$ $0.94868 + 0.94868i, 0.94868 - 0.94868i,$
$-0.94868 + 0.94868i, -0.94868 - 0.94868i$

2) $9x^4 - 4$ $0.8165, -0.8165, 0.8165i, -0.8165i$

3) $x^4 + x2 - 6$ $\sqrt{2}\, i, -\sqrt{2}\, i, \sqrt{8}\, i, -\sqrt{8}\, i$

4) $x^4 + 9x2 + 18$ $\sqrt{3}\, i, -\sqrt{3}\, i, \sqrt{6}\, i, -\sqrt{6}\, i$

N.CN.8, N.CN.9

Comprehension of the Standard

How many roots do the following polynomials have?

1) $x^5 + 4x^3 + x - 1$ 5

2) $x^6 - 1$ 6

3) $2x + 3x^2 + 1$ 2

4) $5x^3 + 6x^4 + 3x$ 4

Assessment Questions

Factor fully:

1) $x^2 + 16 = (x + 4i)(x - 4i)$

2) $x^2 + 36 = (x + 6i)(x - 6i)$

3) $x^2 + 10 = (x + \sqrt{10}\,i)(x - \sqrt{10}\,i)$

4) $x^4 - 9 = (x + 3)(x - 3)(x + 3i)(x - 3i)$

Critical Thinking Homework

Do some research into the Fundamental Theorem of Algebra. Write a report explaining what it is, and where it can be applied. Explain how roots are displayed with real number plots, and explain how this can be extended to complex numbers.

Answers will vary.